

## Synthetic analysis of flexible multibody system including a very flexible body<sup>†</sup>

Ji-Won Yoon<sup>1</sup>, Tae-Won Park<sup>2,\*</sup>, Soo-Ho Lee<sup>1</sup>, Kab-Jin Jun<sup>1</sup> and Sung-Pil Jung<sup>1</sup>

<sup>1</sup>Graduate School of Mechanical Engineering, Ajou University, Suwon, 443-749, Korea

<sup>2</sup>Department of Mechanical Engineering, Ajou University, Suwon, 443-749, Korea

(Manuscript Received December 24, 2008; Revised March 16, 2009; Accepted March 16, 2009)

---

### Abstract

The progress in developing a dynamic analysis solver has different aspects of improvement in the sense of simulating the behavior of the parts. Among them, dynamics in flexible body and large deformable body have been an issue in recent decades. A modal coordinate formulation has been developed and used for analyzing the flexible body dynamics with a commercial dynamic solver, like in ADAMS. Flexible body dynamics using modal coordinates are reliable when the system's deflection is relatively small, and generally its accuracy depends on how many relevant modes are used for the system. Conversely, to simulate the behavior of the large deflected body, absolute nodal coordinate formulation is derived and developed. The theory presents the mixed equations of motion, which consider both the absolute nodal coordinates and absolute cartesian orientation coordinates to simulate the large deflection. Its reliability is proved by many researches and experimental data. In this study, a dynamic solver which can handle the flexible bodies is developed. Three kinds of bodies, rigid, flexible and large deformable body, can be simulated. Its validity is verified by comparison with a commercial analysis program. For further studies, the constraints and force elements between different coordinates will be developed. Solving efficiency would be another major concern to be improved.

*Keywords:* Flexible multibody dynamics; Modal coordinate formulation; Absolute nodal coordinate formulation; Dynamic solver

---

### 1. Introduction

The development of the multibody dynamic solver is passing through different phases of improvement. Among them, flexible multibody dynamics based on structural analysis and its investigation has been a main stream of recent researches, recently [1]. The modal coordinate system has been developed and used for a commercial solver to simulate a flexible multibody dynamic system (MBS) [2, 3]. The method shows good correlation with real-world problems when the deformation is limited to be small. To be reliable, it is necessary to extract realistic modes from

the model and use it for simulation [4]. Many researchers have studied how to select appropriate modes to increase the reliability of a solution [5, 6]. On the other hand, absolute nodal coordinates formulation (ANCF) was initiated in early 2000, and showed improvements in theory and application [7]. The method requires elastic continuum theory derivation with the FEM method and is able to represent nonlinear large deformation [8]. Therefore, it can calculate deflection in both linear and nonlinear ranges. It can be applied to calculate the strain of the cable harness, which suffers from large deflection in the machine system [9]. Fatigue analysis of the support cable used in a bridge is another application. Plastic deformation theory can be added. Many researchers have also reported about various FEM elements for ANCF, such as beam, plate, shell and solid

<sup>†</sup> This paper was presented at the 4th Asian Conference on Multibody Dynamics (ACMD2008), Jeju, Korea, August 20-23, 2008.

\*Corresponding author. Tel.: +82 32 219 2952, Fax.: +82 31 219 1965

E-mail address: park@ajou.ac.kr

© KSME & Springer 2009

[10], and expanding its versatility. In this study, a flexible multibody solver including a very flexible body is developed by using an object-oriented programming language, C++. It is easy to add a module and to develop other subroutines using pre-defined modules.

**2. Numerical analysis flow**

Analysis can be divided into two parts: pre/post processor and solver. At first, the program requests an input file name and creates an output file, which has the same name. Function ‘ReadObject()’ reads the object’s data and distributes them into a dynamic array with initialization process. The input file name traces the path of the location and reads input data from each input data. From them, it initializes simulation data. Variable indexing is also initialized according to the number of bodies, constraints, forces and its coordinate system, which is used for dynamic analysis. To solve the linear system, a sparse solver is used. For solving differential equations, the PECE[6] method is used.

**3. Modal coordinate formulation**

Arbitrary point *i* in a 3-dimensional flexible body, can be described as Eq. (1).

$$\bar{r}_i = \bar{r} + A\bar{S}_i^n = \bar{r} + A(\bar{S}_i^0 n + \bar{u}_i^n) \tag{1}$$

Here, *A* is a transformation matrix,  $\bar{S}_i^0$  is undeformed position vector, and  $\bar{u}_i^n$  is deformation vector. A modal matrix for point *i* has translational and rotational modes as in Eq. (2).

$$\psi^i = [\psi_r^{iT}, \psi_\pi^{iT}] \tag{2}$$

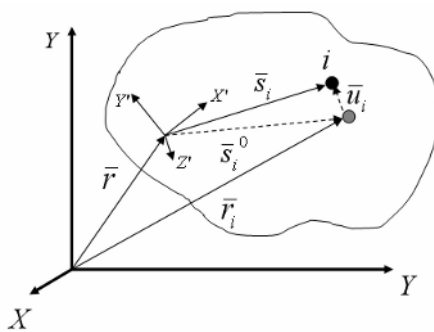


Fig. 1. Coordinate system for flexible body.

A translational displacement is expressed as Eq. (3), which is the combination of modal matrix and modal coordinates.

$$u_i^n = \psi_r^{iT} \bar{a} \tag{3}$$

$\bar{a}$  is a modal position vector. Eqs. (1) and (3) yield Eq. (4).

$$\bar{r}_i = \bar{r} + A(\bar{S}_i^0 + \psi_r^{iT} \bar{a}) \tag{4}$$

This modal information,  $\psi$ , is used for deriving the velocity and the acceleration equation using Eq. (4). According to the virtual work theorem, virtual displacement and rotation can be derived as Eqs.(5) and (6).

$$\delta \bar{r}_i = \delta \bar{r} - A\bar{S}_i^0 \delta \pi^i + A\psi_r^{iT} \delta \bar{a} \tag{5}$$

$$\delta \pi^i = \delta \pi^i + \psi_\pi^{iT} \delta \bar{a} \tag{6}$$

**4. Absolute nodal coordinate formulation**

Shabana[8] proposed an absolute nodal coordinate system, which is useful for analyzing very flexible bodies including nonlinearity effects. The system is derived from continuum theory and finite element method.

A global position vector for ANCF is shown in Eq. (7), which represents arbitrary points in an element *e* of an elastic body *i*. For spatial dynamics, each node has twelve independent coordinates.

$$r^{ie} = S^{ie} (x^{ie}, y^{ie}, z^{ie}) e^{ie}(t) \tag{7}$$

$$e^{ie} = [e^{ie1}, e^{ie2}], e^{ie} = \left[ r, \left( \frac{\partial r}{\partial x} \right)^T, \left( \frac{\partial r}{\partial y} \right)^T, \left( \frac{\partial r}{\partial z} \right)^T \right]^T \tag{8}$$

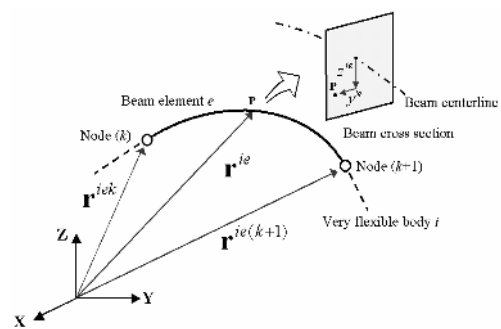


Fig. 2. Beam element in the ANCF.

$r^{ie}$  represents the global position vector and  $\frac{\partial r}{\partial x}$ ,  $\frac{\partial r}{\partial y}$  and  $\frac{\partial r}{\partial z}$  are global position vector gradients at node  $k$ .  $S^{ie}$  is a global shape function matrix.

The beam element is not simplified as a centerline concept, so that rotary inertia effect and torsion can be considered as a real 3D solid model. A global displacement vector can be represented as shown in (7), which means arbitrary position is composed of shape functions, and ANCF coordinates.

Generally, the FEM method uses only 3 translational and 3 rotational coordinates to simulate the behavior of the structure. However, ANCF uses 6 strain vectors to emulate the motion to explain nonlinear large deflection. Therefore, only a few elements are needed than the previous FEM method when meshing. For other elements like plate and shell, there are small variances in the number of nodes and shape function. For instance, 48 generalized coordinates are needed for modeling the plate elements which have 4 nodes at each corner.

**5. Mixed equations of motion**

Eq. (9) is applied to mix three kinds of coordinate systems: Cartesian, modal and ANCF.

$$\begin{bmatrix} M_{rr} & M_{rf} & 0 & \Phi_{q_r}^T \\ M_{fr} & M_{rr} & 0 & \Phi_{q_t}^T \\ 0 & 0 & M_a & \Phi_e^T \\ \Phi_{q_r}^T & \Phi_{q_t}^T & \Phi_e^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_f \\ \ddot{e} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q_r \\ Q_f \\ Q_a \\ \gamma \end{bmatrix} \quad (9)$$

$M_{rr}, M_{rf}, M_{fr}, M_{ff}$  and  $M_a$  are the mass matrix for multibody systems, while  $Q_r, Q_f$  are generalized force vectors. The subscripts r, f, and a indicate Cartesian coordinate, modal coordinate and absolute nodal coordinate, respectively.

**6. Numerical analysis**

**6.1 Flexible body motion**

Both ends of the beam model are restricted by point-constraint. The material properties of the beam are given in Table 1. Exciting force is applied at the mid-point of the beam with  $100\sin(20t)$  N and a vertical displacement of the mid-point is observed.

Table 1. Material properties and dimensions of a beam.

Mass density	$\rho$	$kg/m^3$	7700
Young's Modulus	E	$kgf/m^2$	207e9
Length	L	m	2
Area	A	$m^2$	0.1x0.01

Table 2. Material properties and dimensions of a beam.

Total Mass	m	kg	0.0025
Young's Modulus	E	$kgf/m^2$	200e6
Length	L	m	0.402
Area	A	$M^2$	785.4e-9
Ball	M	Kg	0.02
	$I_{xx}, I_{yy}, I_{zz}$	$Kg/m^3$	1.58e-6

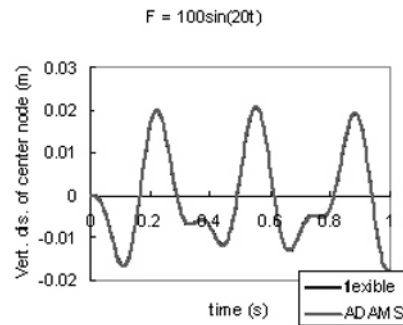


Fig. 3. Vertical displacement of mid-point.

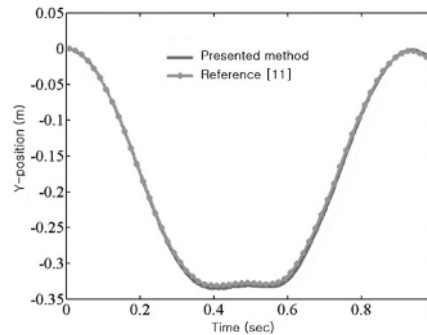


Fig. 4. Vertical displacement of end tip of the beam.

ADAMS, and the developed solver, shows the same results, although it uses normal mode only. It can show that formulation is quite well organized for the developed program.

**6.2 Flexible body motion**

To identify the behavior of a very flexible body, the one-side clamped beam is modeled with 3 beam

elements, attached with 0.02kg ball at the other end. The material properties and dimensions of a beam element are shown in Table 2. Comparison between reference data and results confirms that the results from the ANCF are reliable as shown in Fig. 4.

## 7. Conclusion

In this paper, research trends on flexible body dynamics have been investigated, and 2 types of formulation are derived in a customized dynamic analysis solver to simulate flexible bodies. The solver is developed in C++ language, one of the famous object-oriented programming languages. In deriving flexible body dynamic theory, a modal coordinate system is developed. With the theory of absolute nodal coordinate system, a very flexible body dynamic is also available. With this solver, it is relatively easy to find and freely modify mass, stiffness matrix and detailed information for the system, which are usually difficult to figure out in the commercial dynamic analysis solver. The solver is constructed for easy addition of the new formulation, as well. It has been verified with the commercial analysis program. The developed coordinate systems and solver could be used for the various kinds of flexible body models. For further studies, development of force elements and various kinds of constraint between different coordinate systems will be pursued.

## Acknowledgment

This research was supported by a grant(code 07 next generation high speed railway vehicle A01) from Railroad Technology Development Program (RTDP) funded by Ministry of Land, Transport and Maritime Affairs of Korean government.

## References

- [1] W. S. Yoo, Recent trends in multibody dynamics, *The Korean Society of Mechanical Engineering*, lectures from Spring-Autumn annual conference, (2000) 51-65.
- [2] Ahmed A. Shabana, Flexible Multibody Dynamics: Review of Past and Recent Developments," *Multibody System Dynamics*, Volume 1, Number 2 (1997).
- [3] ADAMS User's manual.
- [4] Computer Aided Design Software Inc, DADS(Dynamic Analysis and Design System) User's manual, U.S.A. (1995).
- [5] R. R. CRAIG, Structural Dynamics, An Introduction to Computer Methods, John Wiley & Sons (1981).
- [6] E. J. HAUG, Computer Aided Kinematics and Dynamics of Mechanical System, Vol. 1 : Basic Method, Prentice-Hall, Inc. (1989).
- [7] S. H. SHIN and W. S. YOO, Effects of Mode Selection, Scaling, and Orthogonalization on the Dynamics Analysis of Flexible Multibody Systems," *J. STRUCT. MECH.*, 21 (4) (1993) 507-507.
- [8] Ahmed. A. Shabana, Computer implementation of the absolute nodal coordinate formulation for flexible multibody dynamics," *Nonlinear Dynamics*, 16 (1998) 293-306.
- [9] O. N. Dmitrochenko, Efficient Simulation of Rigid-Flexible Multibody Dynamics: Some Implementations and Results, *In Proceedings of NATO ASI of Virtual Nonlinear Multibody Systems 1*, W. Schielen and M. Valasek(Eds), Prague, 51-56 (2002).
- [10] W.-S. Yoo, J.-H. Lee, J. -H. Sohn, S.-J. Park, Oleg Dmitrichenko and Dmitri Pogorelov, Large Oscillations of a Thin Cantilever Beam: Physical Experiments and Simulation using Absolute Nodal Coordinate Formulation," *Nonlinear Dynamics*, 34 (1~2) (2003) 3-29.
- [11] J. H. Seo, Dynamic analysis method and its applications for very flexible multibody systems, Dissertation, Ajou University (2005).



**Ji Won Yoon** received B.S. and M.S. degrees in Mechanical Engineering from Ajou University in 2004 and 2006, respectively. Mr. Yoon is currently a Ph.D student at the School of Mechanical Engineering at Ajou University in Suwon, Korea.

He is serving as an instructor for undergraduate students. Mr. Yoon's research interests are in the area of multibody dynamics, flexible body dynamics, and fatigue analysis.